Quantum advantage via non-local games

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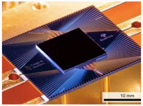
Quantum advantage

Computational advantage of quantum computers:

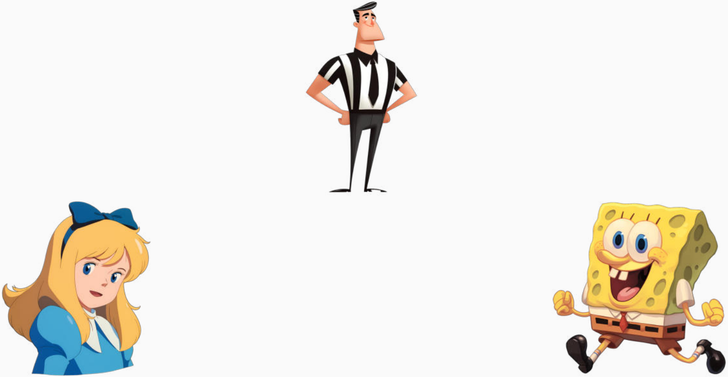
- Sampling from random circuits [A+19]
- Boson sampling [ZDQ+21]
- Quantum error correction below the threshold [A+25]



We focus on a different type of advantage, emphasizing non-locality and entanglement.

It is based on ideas on Bell that have been made precise by Clauser – Horne – Shimony – Holt (CHSH). The theoretical proposal has been verified experimentally by Aspect.

Rules of the CHSH game



Classical vs Quantum strategies

Winning probability:

$$\mathbb{P}(win) = \frac{1}{4}\sum_{x,y}\sum_{a,b} \mathbb{P}(a,b|x,y)\, \mathbb{1}_{a+b=xy}$$

↑ strategy

Deterministic strategies: Alice and Bob compute their answer as a function of their question

$$\mathbb{P}(a,b|x,y) = \mathbb{1}_{a=f_A(x)}\mathbb{1}_{b=f_B(y)}.$$

Classical (or random) strategies: Alice and Bob have access to shared randomness that can be set up before the game starts

$$\mathbb{P}(a,b|x,y) = \sum_{\lambda} p_{\lambda}\mathbb{P}_A(a|x,\lambda)\mathbb{P}_B(b|y,\lambda).$$

Quantum strategies: Alice and Bob share an entangled state

$$\mathbb{P}(a,b|x,y) = \mathrm{Tr}\left[\rho_{AB}\; E_{a|x} \otimes F_{b|y}\right].$$

↑ local measurements ↑

Classical vs Quantum winning probability

CHSH inequality [CHSH69]; Bell's theorem [Bel64]

The best classical strategy wins the game with probability 3/4.

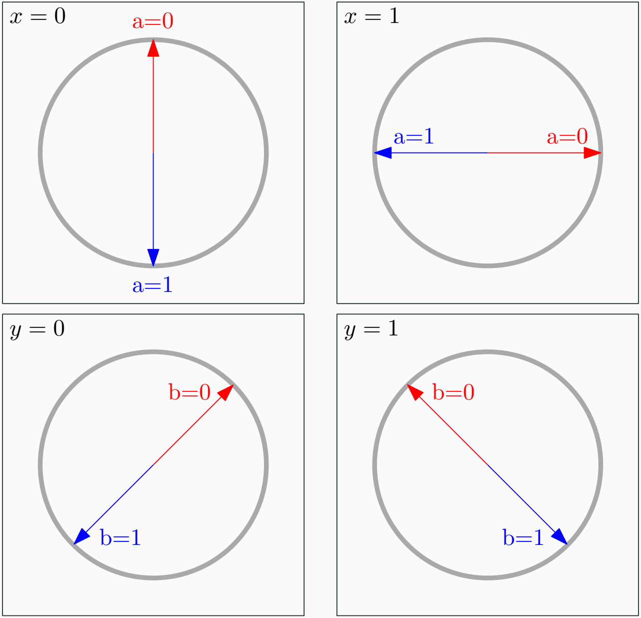
Attained for the (deterministic) strategy $\mathbb{P}(a, b|x, y) = \mathbb{1}_{a=0}\mathbb{1}_{b=0}$.

Quantum mechanics does better!

There exists a quantum strategy that wins with probability $\approx 85\%$.

- Alice and Bob prepare, before the game starts, a maximally entangled state $\rho_{AB} = |\Psi^+\rangle\langle\Psi^+|$, where $|\Psi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$.
- Alice measures the observables $E_0 = Z$ and $E_1 = X$.
- Bob measures the observables $F_0 = (Z + X)/\sqrt{2}$ and $F_1 = (Z - X)/\sqrt{2}$.
- The probability of winning is $\cos^2(\pi/8) = 1/2 + 1/(2\sqrt{2}) \approx 85\%$.

Choice of measurements



Is entanglement necessary?

If Alice and Bob share a separable (i.e. non-entangled) state

probabilities

$$\rho_{AB} = \sum_{\lambda} p_\lambda \, \alpha_\lambda \otimes \beta_\lambda$$

states

their strategy can be written as:

$$\mathbb{P}(a, b|x, y) = \mathrm{Tr}[\rho_{AB} E_{a|x} \otimes F_{b|y}] = \sum_{\lambda} p_\lambda \underbrace{\mathrm{Tr}[\alpha_\lambda E_{a|x}]}_{\mathbb{P}_A(a|x,\lambda)} \underbrace{\mathrm{Tr}[\beta_\lambda F_{b|y}]}_{\mathbb{P}_B(b|y,\lambda)} .$$

Hence, separable states cannot yield a quantum advantage.

However, not all entangled states can provide an advantage.

The Werner state [HQV+17]

$(|01\rangle - |10\rangle)/\sqrt{2}$

$$\rho_W = p|\Psi^-\rangle\langle\Psi^-| + (1-p)\frac{I}{4}$$

is entangled and has a local hidden variable model for $1/3 < p \leq 1/\sqrt{2}$.

Is measurement incompatibility necessary?

Alice has a measurement apparatus $(E_{a|x})_{a=0,1}$ for each question $x = 0, 1$ she receives from the referee. Her measurements are called compatible if there exists another measurement $(G_\lambda)_\lambda$ and probabilities $q(a|x, \lambda)$ s.t.

post-processing

$$\forall a, x \qquad E_{a|x} = \sum_\lambda q(a|x, \lambda) \; G_\lambda \, .$$

single measurement

In this case, the quantum strategy can be written as

$$\mathbb{P}(a, b|x, y) = \sum_\lambda \underbrace{\mathrm{Tr}[\rho_{AB} G_\lambda \otimes I_B]}_{p_\lambda} \underbrace{q(a|x, \lambda)}_{\mathbb{P}_A(a|x,\lambda)} \underbrace{\mathrm{Tr}\left[\frac{\mathrm{Tr}_A[\rho_{AB} G_\lambda \otimes I_B]}{\mathrm{Tr}[\rho_{AB} G_\lambda \otimes I_B]} F_{b|y}\right]}_{\mathbb{P}_B(b|y,\lambda)} .$$

👎 Hence, compatible measurements (for one player) cannot yield a quantum advantage.

◎ Quantitatively, the more incompatible Alice's measurements are, the larger the quantum advantage can be [LN22].

Winning with probability one

[Tsierlson's bound] The quantum strategy that achieves $\approx 85\%$ is optimal.
Hence, using quantum mechanics, it is impossible to produce

$$\mathrm{PR}(a, b|x, y) = \frac{1}{2}\mathbb{1}_{a+b=xy}$$

which wins the game with probability 1. This correlation corresponds to a resource called a Popescu-Rohrlich box [PR94]. PR boxes do not allow communication between Alice and Bob, hence they do not violate faster-than-light communication.

However, PR boxes (and other post-quantum resources) violate other physical or computational principles that "should" be true. ◎ One example is communication complexity: such resources could allow Alice and Bob to compute locally "complicated functions" [BBC+24] and thus collapse communication complexity.

Is the optimal quantum strategy unique?

Self-testing: achieving the maximum probability (≈ 0.85) is a device-independent certificate that the underlying physical system contains a perfect 2-qubit maximally entangled state and the measurements performed on it are the ideal, optimal settings, regardless of the internal workings or complexity of the devices used [ŠB20].

If the maximal winning probability is achieved using a quantum state ρ_{AB} acting on $\mathcal{H}_A \otimes \mathcal{H}_B$ and measurements E, F, resp. on $\mathcal{H}_A$, $\mathcal{H}_B$, then there exist local isometries

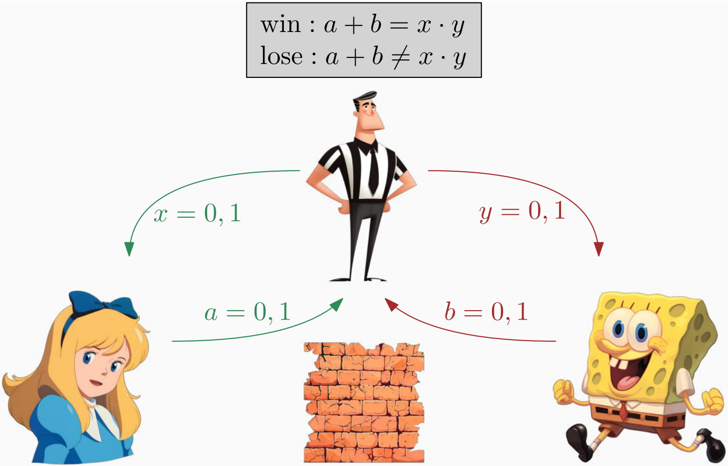
$$V_A : \mathcal{H}_A \to \mathbb{C}^2 \otimes \mathcal{H}_{\mathsf{aux},A} \quad \text{and} \quad V_B : \mathcal{H}_B \to \mathbb{C}^2 \otimes \mathcal{H}_{\mathsf{aux},B}$$

such that

$$\mathrm{Tr}_{\mathcal{H}_{\mathsf{aux},A} \otimes \mathcal{H}_{\mathsf{aux},B}} \left[(V_A \otimes V_B) \rho_{AB} (V_A \otimes V_B)^* \right] = |\Psi^+\rangle\langle\Psi^+|$$

and the restrictions of the measurements VEV^*, VFV^* to the qubit subspaces are unitarily equivalent to the ones in the strategy we presented. ⚠ Robust versions of this theorem exist [GH17].

The take-home slide



$$\mathbb{P}^{\max}_{\text{classical}}(\text{win}) = \frac{3}{4} \quad \text{while} \quad \mathbb{P}^{\max}_{\text{quantum}}(\text{win}) = \frac{1}{2} + \frac{1}{2\sqrt{2}} \approx 85\%$$

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